MATHEMATICS
PART-B
Section - A

* **Answer the following Questions. [Each carries 2 marks] [16]**

1. $y = \dfrac{10^{2x} - 1}{10^{2x} + 1}$

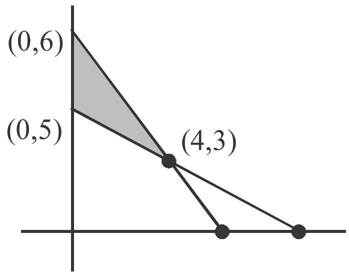
$y(10^{2x}) + y = 10^{2x} - 1$

$10^{2x} = \dfrac{y+1}{1-y}$

$2x = \log_{10}\left(\dfrac{1+y}{1-y}\right)$

$f^{-1}(x) = \dfrac{1}{2}\log_{10}\left(\dfrac{1+x}{1-x}\right)$

2. $x + 2y \ge 10,\ 3x + 4y \le 24$



corner points	z=200x + 500y
(4,3)	2300 - min value
(0,5)	2500
(0,6)	3000

3. $\int e^x\left[\dfrac{x^2-4}{(x+2)^2} + \dfrac{4}{(x+2)^2}\right]dx$

$\int e^x\left[\dfrac{x-2}{x+2} + \dfrac{4}{(x+2)^2}\right]dx = e^x\left(\dfrac{x-2}{x+2}\right) + C$

4. $(1+y^2)\,dx = (\tan^{-1}y - x)\,dy$

$\dfrac{dx}{dy} + \dfrac{x}{1+y^2} = \dfrac{\tan^{-1}y}{1+y^2}$

I.F. $= e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$

MATHEMATICS
PART-B
Section - A

* **નીચેના પ્રશ્નોના જવાબ આપો. [દરેકના બે ગુણ] [16]**

1. $y = \dfrac{10^{2x} - 1}{10^{2x} + 1}$

$y(10^{2x}) + y = 10^{2x} - 1$

$10^{2x} = \dfrac{y+1}{1-y}$

$2x = \log_{10}\left(\dfrac{1+y}{1-y}\right)$

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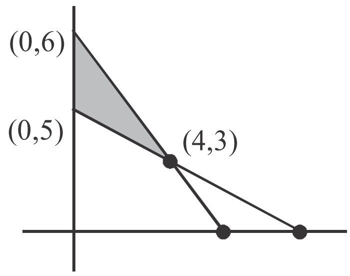
2. $x + 2y \ge 10,\ 3x + 4y \le 24$



શિરોબિંદુઓ	z=200x + 500y
(4,3)	2300 - ન્યુનતમ કિંમત
(0,5)	2500
(0,6)	3000

3. $\int e^x\left[\dfrac{x^2-4}{(x+2)^2} + \dfrac{4}{(x+2)^2}\right]dx$

$\int e^x\left[\dfrac{x-2}{x+2} + \dfrac{4}{(x+2)^2}\right]dx = e^x\left(\dfrac{x-2}{x+2}\right) + C$

4. $(1+y^2)\,dx = (\tan^{-1}y - x)\,dy$

$\dfrac{dx}{dy} + \dfrac{x}{1+y^2} = \dfrac{\tan^{-1}y}{1+y^2}$

I.F. $= e^{\int \frac{1}{1+y^2}dx} = e^{\tan^{-1}y}$

$$e^{\tan^{-1}y}x = \int e^{\tan^{-1}y}\frac{\tan^{-1}y}{1+y^2}dy$$

$$\Rightarrow \quad xe^{\tan^{-1}y} = \int te^t dt$$

$$\Rightarrow \quad xe^{\tan^{-1}y} = e^{\tan^{-1}y}(\tan^{-1}y - 1) + c$$

5. $\bar{a}, \bar{b}, \bar{c} \neq \bar{o},\ \bar{a}\times\bar{b} = \bar{c},\ \bar{b}\times\bar{c} = \bar{a}$

$\bar{b}\times(\bar{a}\times\vec{b}) = \bar{a}$

$(\bar{b}\cdot\bar{b})\bar{a} - (\bar{b}\cdot\bar{a})\bar{b} = \bar{a}$

$|\bar{b}|^2\ \bar{a} - (\bar{b}\cdot\bar{a})\bar{b} = \bar{a}$

$|\bar{b}|^2 = 1$

$|\bar{b}| = 1$

6. $\ell + m + n = 0,\ \ell^2 - m^2 + n^2 = 0$

$\ell^2 + n^2 = m^2$

$\ell^2 + n^2 = m^2$

$\ell^2 + n^2 = (-\ell - n)^2 = \ell^2 + n^2 + 2\ell n$

$2\ell n = 0$

$\ell = 0$ or $n = 0$

$m = -n$ $\qquad \ell = -m$

$\vec{\ell} = (0, -n, n)$ $\qquad \vec{m} = (-m, m, 0)$

$$\cos\theta = \frac{|\vec{\ell}\cdot\vec{m}|}{|\vec{\ell}||\vec{m}|} = \frac{|0 - mn|}{|\sqrt{2}n\cdot\sqrt{2}m|} = \frac{1}{2}$$

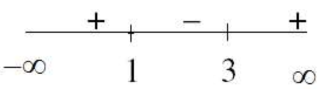
$$\theta = \frac{\pi}{3}$$

7. $f(x) = 2x^3 - 12x^2 + 18x + 15$

$f'(x) = 6x^2 - 24x + 18$

$= 6(x^2 - 4x + 3)$

$= 6(x - 3)(x - 1)$



$(-\infty, 1)$ & $(3, \infty) \rightarrow$ increasing $(1,3) \rightarrow$ decreasing

$$e^{\tan^{-1}y}x = \int e^{\tan^{-1}y}\frac{\tan^{-1}y}{1+y^2}dy$$

$$\Rightarrow \quad xe^{\tan^{-1}y} = \int te^t dt$$

$$\Rightarrow \quad xe^{\tan^{-1}y} = e^{\tan^{-1}y}(\tan^{-1}y - 1) + c$$

5. $\bar{a}, \bar{b}, \bar{c} \neq \bar{o},\ \bar{a}\times\bar{b} = \bar{c},\ \bar{b}\times\bar{c} = \bar{a}$

$\bar{b}\times(\bar{a}\times\vec{b}) = \bar{a}$

$(\bar{b}\cdot\bar{b})\bar{a} - (\bar{b}\cdot\bar{a})\bar{b} = \bar{a}$

$|\bar{b}|^2\ \bar{a} - (\bar{b}\cdot\bar{a})\bar{b} = \bar{a}$

$|\bar{b}|^2 = 1$

$|\bar{b}| = 1$

6. $\ell + m + n = 0,\ \ell^2 - m^2 + n^2 = 0$

$\ell^2 + n^2 = m^2$

$\ell^2 + n^2 = m^2$

$\ell^2 + n^2 = (-\ell - n)^2 = \ell^2 + n^2 + 2\ell n$

$2\ell n = 0$

$\ell = 0$ or $n = 0$

$m = -n$ $\qquad \ell = -m$

$\vec{\ell} = (0, -n, n)$ $\qquad \vec{m} = (-m, m, 0)$

$$\cos\theta = \frac{|\vec{\ell}\cdot\vec{m}|}{|\vec{\ell}||\vec{m}|} = \frac{|0 - mn|}{|\sqrt{2}n\cdot\sqrt{2}m|} = \frac{1}{2}$$

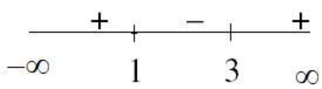
$$\theta = \frac{\pi}{3}$$

7. $f(x) = 2x^3 - 12x^2 + 18x + 15$

$f'(x) = 6x^2 - 24x + 18$

$= 6(x^2 - 4x + 3)$

$= 6(x - 3)(x - 1)$



$(-\infty, 1)$ & $(3, \infty) \rightarrow$ વધતું વિધેય $(1,3) \rightarrow$ ઘટતું વિધેય

8. $= \begin{vmatrix} x & 0 & 1 \\ -y & 2y & 1 \\ 0 & -2z & 1+3z \end{vmatrix} C_{21}(-1), C_{32}(-1)$

$= x[2y + 6yz + 2z] + (2yz)$

$= 2xy + 6xyz + 2xz + 2yz$

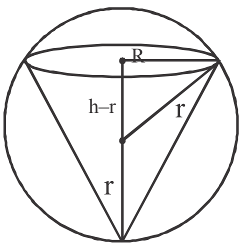
$= 2xyz\left(3 + \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$

Section - B

* **Answer the following Questions. [Each carries 3 marks] [18]**

9. $r^2 = R^2 + (h-r)^2$

$R^2 = r^2 - (h-r)^2$



$R^2 = 2hr - h^2$

$V = \frac{1}{3}\pi R^2 h$

$= \frac{\pi}{3}[2hr - h^2]h$

$f(h) = \frac{\pi}{3}(2h^2 r - h^3)$

$f'(h) = \frac{\pi}{3}(4hr - 3h^2) = 0$

$4r - 3h = 0$

$h = \frac{4r}{3}$

$f''(h) = \frac{\pi}{3}(4r - 6h)$

$= \frac{\pi}{3}[4r - 6\left(\frac{4r}{3}\right) < 0$

So that volume is maximum

8. $= \begin{vmatrix} x & 0 & 1 \\ -y & 2y & 1 \\ 0 & -2z & 1+3z \end{vmatrix} C_{21}(-1), C_{32}(-1)$

$= x[2y + 6yz + 2z] + (2yz)$

$= 2xy + 6xyz + 2xz + 2yz$

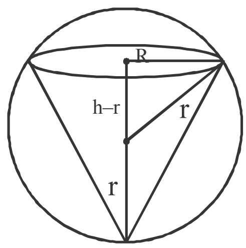
$= 2xyz\left(3 + \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$

Section - B

* **નીચેના પ્રશ્નોના જવાબ આપો. [દરેકના ત્રણ ગુણ] [18]**

9. $r^2 = R^2 + (h-r)^2$

$R^2 = r^2 - (h-r)^2$



$R^2 = 2hr - h^2$

$V = \frac{1}{3}\pi R^2 h$

$= \frac{\pi}{3}[2hr - h^2]h$

$f(h) = \frac{\pi}{3}(2h^2 r - h^3)$

$f'(h) = \frac{\pi}{3}(4hr - 3h^2) = 0$

$4r - 3h = 0$

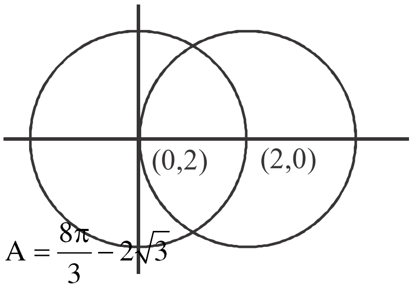
$h = \frac{4r}{3}$

$f''(h) = \frac{\pi}{3}(4r - 6h)$

$= \frac{\pi}{3}[4r - 6\left(\frac{4r}{3}\right) < 0$

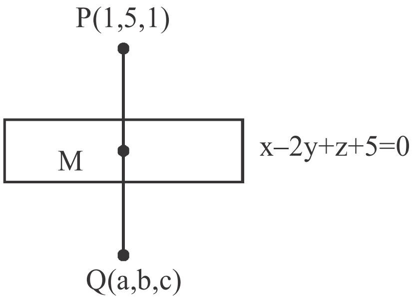
તેથી સાબિત થાય છે કે ઘનફળ મહત્તમ છે.

10. $A = 4\left[\int_1^2 \sqrt{4-x^2}\,dx\right]$



$A = \frac{8\pi}{3} - 2\sqrt{3}$

11.



equation of $\overleftrightarrow{PM} : \bar{r} = (1,5,1) + k(1,-2,1)$

M = (1 + k, 5 – 2k, 1 + k)

- M is on plane

1 + k – 10 + 4k + 1 + k + 5 = 0

$6k = 3 \quad \Rightarrow k = \frac{1}{2}$

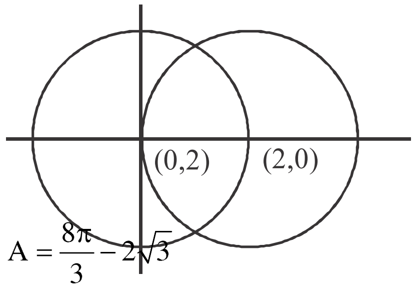
So $M\left(\frac{3}{2}, 4, \frac{3}{2}\right)$

Then, Q(2,3,2)

12. Let $\cos^{-1}\frac{a}{b} = \theta \Rightarrow \left(\cos\theta = \frac{a}{b}\right)$

$\Rightarrow \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) + \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$

10. $A = 4\left[\int_1^2 \sqrt{4-x^2}\,dx\right]$



$A = \frac{8\pi}{3} - 2\sqrt{3}$

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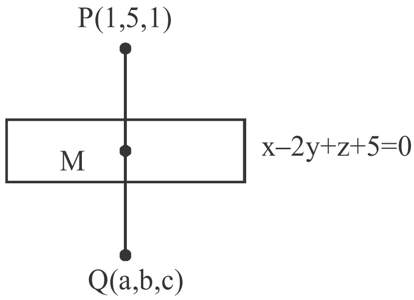

$\overleftrightarrow{PM}$: નું સમીકરણ $\bar{r} = (1,5,1) + k(1,-2,1)$

M = (1 + k, 5 – 2k, 1 + k)

- M એ સમતલ પર છે.

1 + k – 10 + 4k + 1 + k + 5 = 0

$6k = 3 \quad \Rightarrow k = \frac{1}{2}$

So $M\left(\frac{3}{2}, 4, \frac{3}{2}\right)$

તેથી, Q(2,3,2)

12. Let $\cos^{-1}\frac{a}{b} = \theta \Rightarrow \left(\cos\theta = \frac{a}{b}\right)$

$\Rightarrow \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) + \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$

$$=\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}+\frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}$$

$$=2\sec\theta=\frac{2b}{a}$$

13. Let D_1 = The event that selected customer is man

D_2 = selected customer is woman

$$P(D_1)=\frac{6}{10}\quad P(D_2)=\frac{4}{10}$$

C = Selected customers orders dish A

$$P(C/D_1)=\frac{8}{10},\ P(C/D_2)=\frac{3}{10}$$

$$P(C)=\frac{6}{10}\times\frac{8}{10}+\frac{4}{10}\times\frac{3}{10}=\frac{60}{100}$$

Required ration = 60 : 40 = 3 : 2

14. $A^{-1}\begin{bmatrix}\frac{1}{2} & \frac{-3}{2} & \frac{1}{2}\\ -4 & 3 & -1\\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2}\end{bmatrix}$

Section - C

* **Answer the following Questions. [Each carries 4 marks] [16]**

15. According to newton's law $\frac{dT}{dt}\propto(T-S)$

$$\frac{dT}{dt}=-K(T-S)$$

log|T – S| = Kt + C

Now, t = 0 $\Rightarrow$ T = 100°F

log |100 – S| = C

So, log [T – S) =–Kt + log [100–S)

Now, t = 5 $\Rightarrow$ T = 50°F

log(50 – S) = –5k + log(100–S) ....(1)

log (40 – S) = –10k + log(100 – S)

$$=\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}+\frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}$$

$$=2\sec\theta=\frac{2b}{a}$$

13. ધારો કે D_1 = પસંદ કરેલ ગ્રાહક પુરુષ હોય તે ઘટના

D_2 = પસંદ કરેલ ગ્રાહક સ્ત્રી હોય તે ઘટના

$$P(D_1)=\frac{6}{10}\quad P(D_2)=\frac{4}{10}$$

C = પસંદ કરેલ ગ્રાહક A પ્રકારની ડીશનો ઓર્ડર કરે છે.

$$P(C/D_1)=\frac{8}{10},\ P(C/D_2)=\frac{3}{10}$$

$$P(C)=\frac{6}{10}\times\frac{8}{10}+\frac{4}{10}\times\frac{3}{10}=\frac{60}{100}$$

જરૂરી ગુણોત્તર = 60 : 40 = 3 : 2

14. $A^{-1}\begin{bmatrix}\frac{1}{2} & \frac{-3}{2} & \frac{1}{2}\\ -4 & 3 & -1\\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2}\end{bmatrix}$

Section - C

* **નીચેના પ્રશ્નોના જવાબ આપો. [દરેકના ચાર ગુણ] [16]**

15. ન્યુટનના નિયમ મુજબ $\frac{dT}{dt}\propto(T-S)$

$$\frac{dT}{dt}=-K(T-S)$$

log|T – S| = Kt + C

Now, t = 0 $\Rightarrow$ T = 100°F

log |100 – S| = C

So, log [T – S) =–Kt + log [100–S)

Now, t = 5 $\Rightarrow$ T = 50°F

log(50 – S) = –5k + log(100–S) ....(1)

log (40 – S) = –10k + log(100 – S)

...(2)

$$\frac{1}{5}\log\left(\frac{50-S}{100-S}\right)=\frac{1}{10}\log\frac{40-S}{100-S}$$

$$S=\frac{75}{2}=37.5^{o}F$$

16. $\int_0^1 \tan^{-1}\left(\frac{x+1-x}{1-x(1-x)}\right)$

$$=\int_0^1 \tan^{-1}x\,dx+\int_0^1 \tan^{-1}(1-x)dx$$

$$=\int_0^1 \tan^{-1}xdx+\int_0^1 \tan^{-1}xdx$$

$$=2\int_0^1 \tan^{-1}xdx$$

$$=2\left[(x\tan^{-1}x)_0^1-\frac{1}{2}\int_0^1\frac{2x}{1+x^2}dx\right]$$

$$=2\left[\frac{\pi}{4}\right]-[\log(1+x^2)]_0^1$$

$$=\frac{\pi}{2}-\log 2$$

17. $\frac{1}{2}\int\frac{2}{x^4+1}dx=\frac{1}{2}\left[\int\frac{1+x^2}{1+x^4}dx+\int\frac{1-x^2}{1+x^4}dx\right]$

$$\left[\int\frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2}dx-\int\frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-2}dx\right]$$

$$=\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)-\frac{1}{4\sqrt{2}}\log\left|\frac{x^2+1-\sqrt{2}x}{x^2+1+\sqrt{2}x}\right|+C$$

...(2)

$$\frac{1}{5}\log\left(\frac{50-S}{100-S}\right)=\frac{1}{10}\log\frac{40-S}{100-S}$$

$$S=\frac{75}{2}=37.5^{o}F$$

16. $\int_0^1 \tan^{-1}\left(\frac{x+1-x}{1-x(1-x)}\right)$

$$=\int_0^1 \tan^{-1}x\,dx+\int_0^1 \tan^{-1}(1-x)dx$$

$$=\int_0^1 \tan^{-1}xdx+\int_0^1 \tan^{-1}xdx$$

$$=2\int_0^1 \tan^{-1}xdx$$

$$=2\left[(x\tan^{-1}x)_0^1-\frac{1}{2}\int_0^1\frac{2x}{1+x^2}dx\right]$$

$$=2\left[\frac{\pi}{4}\right]-[\log(1+x^2)]_0^1$$

$$=\frac{\pi}{2}-\log 2$$

17. $\frac{1}{2}\int\frac{2}{x^4+1}dx=\frac{1}{2}\left[\int\frac{1+x^2}{1+x^4}dx+\int\frac{1-x^2}{1+x^4}dx\right]$

$$\left[\int\frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2}dx-\int\frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-2}dx\right]$$

$$=\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)-\frac{1}{4\sqrt{2}}\log\left|\frac{x^2+1-\sqrt{2}x}{x^2+1+\sqrt{2}x}\right|+C$$

18. $y = x[\log x - \log(a + bx)]$

$$y_1 = [\log x - \log(a + bx)] + x\left[\frac{1}{x} - \frac{b}{a+bx}\right]$$

$$y_1 = \frac{y}{x} + \frac{a}{a+bx}$$

$$xy_1 - y = \frac{ax}{a+bx} \quad ....(1)$$

Again differenciate w.r. to 'x' we get

$$\rightarrow xy_2 + y_1 - y_1 = \frac{a(a+bx) - axb}{(a+bx)^2}$$

$$xy_2 = \frac{a^2}{(a+bx)^2}$$

From equation (1)
multiplying by x^2

$$x^3y_2 = \frac{a^2x^2}{(a+bx)^2}$$

$$x^3y_2 = (xy_1 - y)^2$$

18. $y = x[\log x - \log(a + bx)]$

$$y_1 = [\log x - \log(a + bx)] + x\left[\frac{1}{x} - \frac{b}{a+bx}\right]$$

$$y_1 = \frac{y}{x} + \frac{a}{a+bx}$$

$$xy_1 - y = \frac{ax}{a+bx} \quad ....(1)$$

'x' ની સાપેક્ષે ફરી વખત વિકલન કરતા

$$\rightarrow xy_2 + y_1 - y_1 = \frac{a(a+bx) - axb}{(a+bx)^2}$$

$$xy_2 = \frac{a^2}{(a+bx)^2}$$

સમીકરણ (1) પરથી
x^2 વડે ગુણાકાર કરતા

$$x^3y_2 = \frac{a^2x^2}{(a+bx)^2}$$

$$x^3y_2 = (xy_1 - y)^2$$